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STRUCTURAL INELASTICITY XVI

Elastic-Plastic Plate with Arbitrary Poisson's Ratio

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ELASTIC-PLASTIC PLATE WITH

ARBITRARY POISSON'S RATIO[1]

By

Philip G. Hodge, Jr.[2]

Abstract

An earlier solution for the bending of a built-in circular plate made of an incompressible elastic/perfectly-plastic material is extended to include any value of Poisson's ratio ν. Qualitative differences in the nature of the solution are observed for $0 \leq \nu < 1/5$, $1/5 < \nu < 1/3$, and $1/3 < \nu \leq 1/2$.

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ELASTIC-PLASTIC PLATE WITH ARBITRARY POISSON'S RATIO

Some twenty years ago Tekinalp [1] published a paper on the elastic-plastic bending of a built-in circular plate under a slowly increasing uniform monotonic pressure. He assumed that the plate material was incompressible in both the elastic and plastic phases, and obtained a complete solution almost, but not quite, up to the yield-point load.

The purpose of the present note is to point out that the solution presents a rather interesting behavior variation with Poisson's ratio ν. In particular, for any $\nu \leq 1/3$, a complete solution is obtained all the way to the yield-point load, so that the unrealistic approximation of elastic incompressibility actually adds to the complexity of the problem, rather than simplifying it as in the case of plane strain.

We consider a plate of radius A made of an elastic/perfectly-plastic material which satisfies Tresca's yield criterion

$$\max[\,|m_r|,\ |m_\phi|,\ |m_r - m_\phi|\,] \leq 1 \qquad (1)$$

where $m_i = M_i/M_0$, M_0 being the yield moment. The moments must satisfy the equilibrium equation

$$(rm_r)' - m_\phi = -3pr^2 \qquad (2)$$

where $p = PA^2/6M_0$ and $r = R/A$.

The elastic solution may be written [2]

$$m_r = C/r^2 + D - 3(3+\nu)pr^2/8$$
$$m_\phi = -C/r^2 + D - 3(1+3\nu)pr^2/8 \qquad (3)$$

For p sufficiently small, stage 1, the plate is fully elastic with

$$C = 0 \qquad D = 3(1+\nu)p/8 \tag{4}$$

For all values of ν, stage 1L, the limit of stage 1, occurs when $m_r(1) = -1$ at

$$p_1 = 4/3 \tag{5}$$

Figure 1 shows the resulting stress profile for two different values of ν.

In stage 2, the plate is still fully elastic in the interior, but there is a hinge circle at $r = 1$. Thus the boundary condition $w'(1) = 0$ of stage 1 is replaced by $m_r(1) = -1$. This condition leads to

$$C = 0 \qquad D = -1 + 3(3+\nu)p/8 \tag{6}$$

Stage 2 will terminate when $m_\phi(1) = 0$ or when $m_r(0) = m_\phi(0) = 1$, whichever happens first. For $\nu > 1/5$ stage 2L is characterized by the $r = 0$ end of the stress profile reaching B (Fig. 1a) with

$$p_2 = 16/[3(3+\nu)] \qquad \nu > 1/5 \tag{7}$$

For smaller ν, the $r = 1$ end of the stress profile reaches D first (Fig. 1b) with

$$p_2 = 4/[3(1-\nu)] \tag{8}$$

Evidently stage 3 will take two different forms depending upon the value of ν. For $\nu > 1/5$, the central part of the plate is on side BC, Fig. 1a, with

$$m_r = 1 - pr^2 \qquad m_\phi = 1 \qquad 0 \le r \le \eta \tag{9}$$

The outer part is still elastic with

$$C = -(1+3\nu)p\eta^4/8 \qquad D = 1 + (1+3\nu)p\eta^2/4 \qquad (10)$$

where the load p is given in terms of the elastic-plastic boundary by

$$p = 16[3(3+\nu) - 2(1+3\nu)\eta^2 + (1+3\nu)\eta^4]^{-1} \qquad (11)$$

For stage 3L, $m_\phi(1) = 0$ which leads to

$$p = [8/(1+3\nu)][3-2\eta^2 - \eta^4]^{-1} \qquad (12)$$

Combining (11) and (12) we see that

$$\eta^2 = \frac{1}{3}\left[2\sqrt{\frac{2(6\nu-1)}{1+3\ \nu}} - 1\right] \qquad \nu > 1/5 \qquad (13)$$

whence p_3 is obtained from either (11) or (12).

If $\nu < 1/5$, stage 3 will be partially on side CD, Fig. 1b, with

$$m_\phi = \log r + (3p/2)(1-r^2) \qquad m_r = m_\phi - 1 \qquad \xi \leq r \leq 1 \qquad (14)$$

whereas the center of the plate is elastic with

$$C = 0 \qquad D = (3/2)(p-1) + \log \xi \qquad (15)$$

and the pressure is given in terms of the elastic-plastic boundary ξ by

$$p = 4[3(1-\nu)\xi^2]^{-1} \qquad (16)$$

Stage 3 will terminate when $m_r(0) = 1$ whence

$$3p - 5 + \log \xi^2 = 0 \qquad \nu < 1/5 \qquad (17)$$

Combination of (16) and (17) leads to a transcendental equation for p_3 which is easily solved numerically for any given ν.

For any ν, stage 4 will consist of a center region $0 \leq r \leq \eta$ which is plastic on side BC, a plastic annulus

$\xi \leq r \leq 1$ on side CD, and an elastic annulus in between. Moments for the three regions are defined by Eqs. (9), (3), and (14), respectively. Continuity conditions at $r = \eta$ again produce C and D as given by (10), whence continuity conditions at $r = \xi$ lead to

$$p = 2\frac{3-2\log\xi}{6-(1+3\nu)\eta^2-3(1-\nu)\xi^2} \tag{18a}$$

$$p = \frac{4\xi^2}{(1+3\nu)\eta^4+3(1-\nu)\xi^4} \tag{18b}$$

which define the boundaries η and ξ implicitly in terms of p.

Stage 4 will terminate when either η or ξ reach point C, Fig. 1, whichever happens first. If η reaches C first, then $m_\phi(\eta) = 0$ whence

$$p\eta^2 = 1 \tag{19}$$

Equations (18) and (19) define ξ, η, and p_4 at stage 4L. In particular, it follows from (18b) and (19) that

$$(\xi^2-\eta^2)[3(1-\nu)\xi^2-(1+3\nu)\eta^2] = 0 \tag{20}$$

Since $\eta < \xi$, Eq. (20) is meaningful if and only if $\nu > 1/3$, and

$$\eta^2 = 3\frac{1-\nu}{1+3\nu}\xi^2 \tag{21}$$

Substituting (21) in (18) and eliminating p we see that ξ^2 must satisfy the transcendental equation

$$\frac{1+3\nu}{1-\nu} + \xi^2\log\xi^2 - (4+3\nu)\xi^2 = 0 \tag{22}$$

which is easily solved numerically for any given ν. Then η^2 and p_4 are given by (21) and (19), respectively.

If ξ reaches C first, then $m_\phi(\xi) = 0$ whence

$$p = \frac{2}{3}\,\frac{1-\log \xi}{1-\xi^2} \qquad (23)$$

Combining Eqs. (23) and (18a) we can write the result

$$y = \frac{v-6u+3 \log u + 12}{v + 2 + \log u} \qquad (24)$$

where we have defined

$$u = 1/\xi^2 \qquad y = \eta^2/\xi^2 \qquad v = 3\nu(2 + \log u) \qquad (25)$$

Similarly, from (23) and (18b) we obtain

$$y^2 = \frac{v+3(4u-6-\log u}{v + 2 + \log u} \qquad (26)$$

Then, eliminating y between (24) and (26) we can write the result as

$$(3u-5-\log u)[2v-3(u-3-\log u)] = 0 \qquad (27)$$

Finally substituting the two roots of (27) in (24) we obtain either $y = 1$ or $y = -3$. Thus the second root is physically meaningless and the first one predicts $\xi = \eta$.

Therefore, for $\nu \leq 1/3$, η and ξ reach C simultaneously at a load defined by

$$5 + \log p - 3p = 0 \qquad \xi = \eta = 1/\sqrt{p} \qquad \nu \leq 1/3 \quad (28)$$

In this case 4L is the yield point load solution, so that the problem is complete and p_4 is the yield-point load.

For all of the stages considered so far, each stress point has either remained elastic or gone directly into its only plastic regime. Under this state of "regular progression" [3], the flow law may be integrated with respect to time. Therefore, for a point on BC the slope is

$$w' = B - (1-\nu)r + pr^3/3 \qquad (29a)$$

for an elastic point it is

$$w' = (1+\nu)C/r - (1-\nu)Dr + 3(1-\nu^2)pr^3/8 \qquad (29b)$$

and for a point on CD it is

$$w' = A/r + (1-\nu)[r \log r + 3p(2r-r^3)/4] \qquad (29c)$$

with C and D known, there are sufficient continuity conditions to find A and B, so that slopes are readily determined. A further integration of Eqs. (29) with continuity and the boundary condition $w(1) = 0$ provides the deflection for all stages to date. Thus a complete solution is available for $\nu < 1/3$.

For $\nu > 1/3$, a stage 5 is necessary before the yield-point load is reached. Equations are easily found for the four regimes

$$
\begin{array}{ll}
0 \le r \le \eta & \text{side BC} \\
\eta \le r \le \zeta & \text{side CD} \\
\zeta \le r \le \xi & \text{elastic} \\
\xi \le r \le 1 & \text{side CD}
\end{array}
\qquad (29)
$$

Continuity conditions would provide three equations for implicit determination of η, ζ , and ξ in terms of p. In particular, Eq. (19) would still hold, showing that as p increased, η would decrease. Consider then, a material point which was overtaken by the advancing elastic-plastic boundary η near the end of stage 4 and hence is on side BC at stage 4L. Early in stage 5 this point will be passed over again by the retreating boundary η and hence will enter regime CD. For

such irregular progression involving motion from one regime to another, the flow law cannot be integrated directly, so that our solution, as with Tekinalp's [1] for $\nu = 1/2$, is left incomplete for all $\nu > 1/3$.

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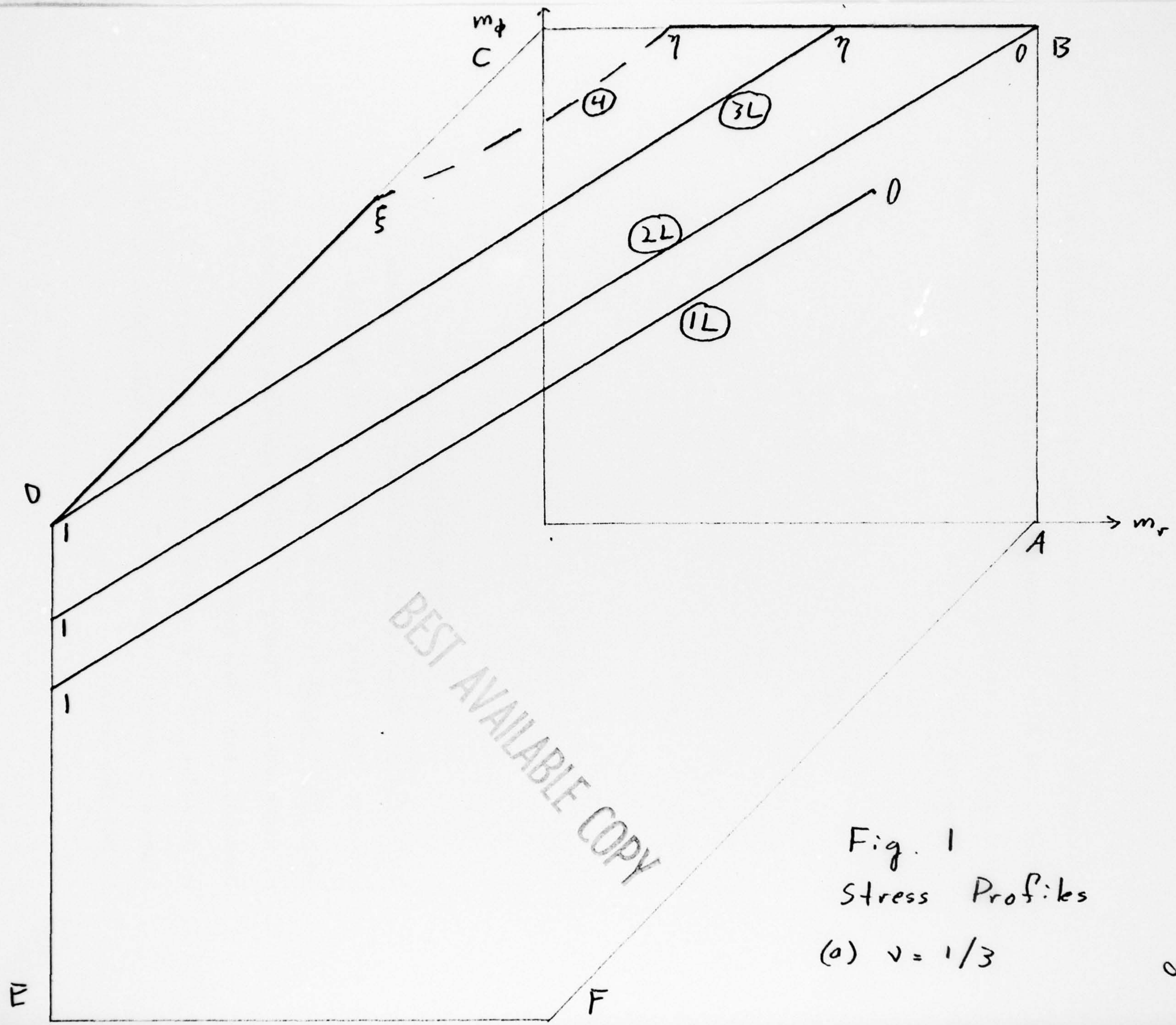

Fig. 1
Stress Profiles
(a) $\nu = 1/3$

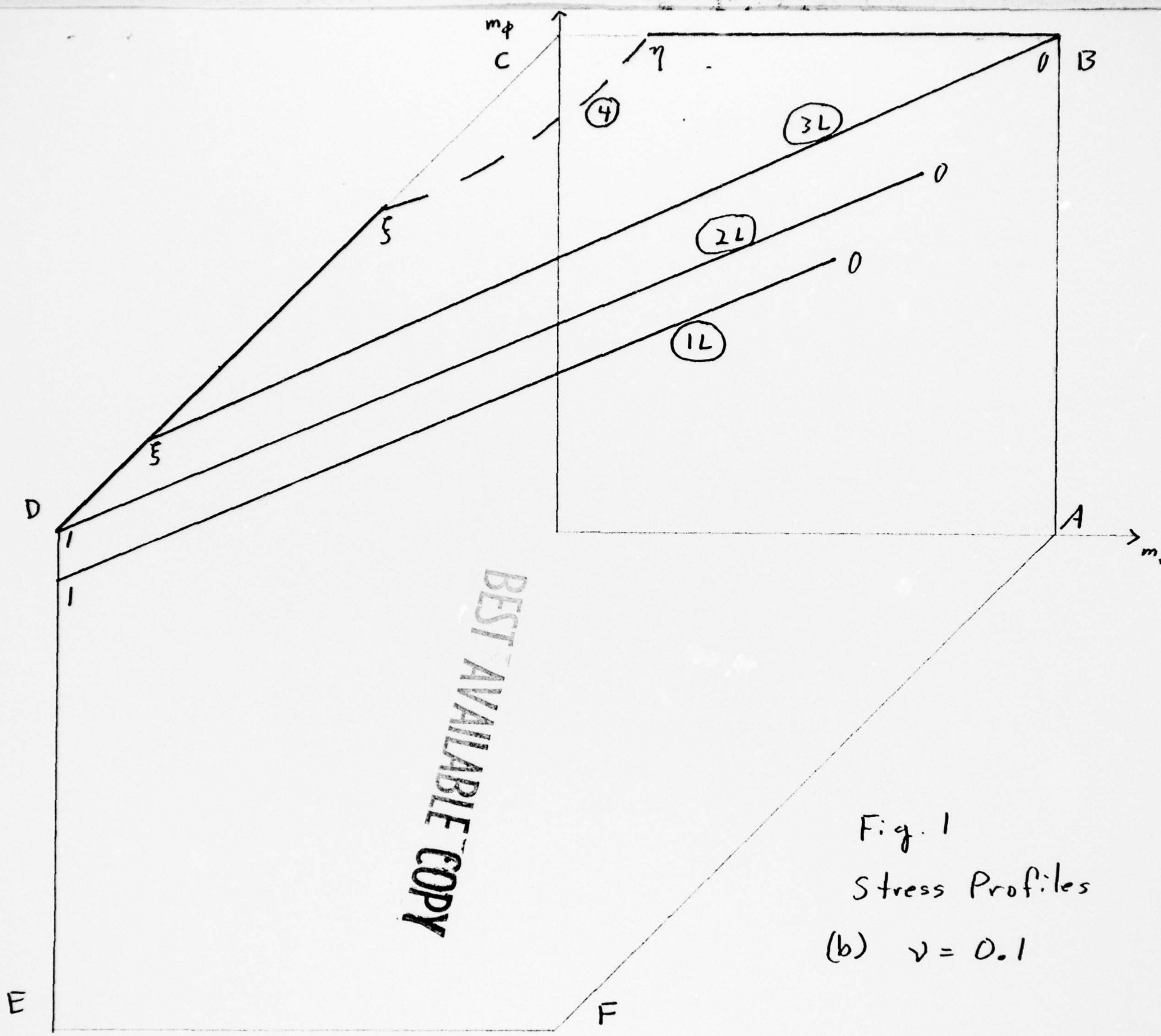

Fig. 1
Stress Profiles
(b) $\nu = 0.1$